Negative Mass?

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Introduction

The purpose of this is for me to try writing a little article about something of interest to me. A short listen to a totally sane Joe Rogan podcast with Eric Weinstein got me thinking about some funny scify physics among which was antigravity and and negative mass. A small remark which was made was that if we a negative mass would chase a postive mass for as long as they don't collide with a planet, or something big. That got me thinking, and I wanted to derive this for myself, which i did. My goal is to explain this and derive it here.

So lets begin with stating Newtons law of universal Gravitation and the definition of force,

$$F = \frac{Gm_1m_2}{d^2}$$

$$F = ma$$

Here G is the Gravitation constant, m_1, m_2 are two masses and d is the distance between them. We should all be familiar with these and if not you can take them for granted.

Derivation

In order to gain more information from the two equations in the introduction we will modify both slightly. If we let m_1 and x_1 be the mass and position on a line of particle 1. Similarly with m_2 and x_2 for particle 2. We can modify these equations to give us the direction of the force applied to a given particle

$$F_1 = \frac{Gm_1m_2}{|x_2 - x_1|^3}(x_2 - x_1) = \frac{Gm_1m_2}{|x_2 - x_1|^2}\frac{(x_2 - x_1)}{|x_2 - x_1|}$$

$$F_2 = \frac{Gm_1m_2}{|x_2 - x_1|^3}(x_2 - x_1) = \frac{Gm_1m_2}{|x_2 - x_1|^2}\frac{(x_2 - x_1)}{|x_2 - x_1|}$$

The equations above tell us the force on each particle, for clarity, we will only be considering the 2 particles. The term $\frac{(x_2 - x_1)}{|x_2 - x_1|}$ may remind you of a unit vector, which it is, but in one dimension Its purpose is to change the sign of the force to point towards the position of the other particle. Now using $F_1 = m_1a_1$ and $F_2 = m_2a_2$ we can get the acceleration of one particle as a result of the force created by another particle.

$$F_1 = m_1a_1 = \frac{Gm_1m_2}{|x_2 - x_1|^3}(x_2 - x_1)$$

$$F_2 = m_2a_2 = \frac{Gm_1m_2}{|x_1 - x_2|^3}(x_1 - x_2)$$

Simplifying the above we get the following,

$$a_1 = \frac{Gm_2}{|x_2 - x_1|^3}(x_2 - x_1)$$

$$a_2 = \frac{Gm_1}{|x_1 - x_2|^3}(x_1 - x_2)$$

Notice that the left most term on the right hand side of both equations is going to stay constant so to make it a bit easier to read we let C be a positive constant so that,

$$C = \frac{G}{|x_2 - x_1|^3} = \frac{G}{|x_1 - x_2|^3}$$

Then,

$$a_1 = Cm_2(x_2 - x_1)$$

$$a_2 = Cm_1(x_1 - x_2)$$

Now the two equations look very similar and are different by a minus sign and mass so

$$\frac{a_1}{Cm_2} = (x_2 - x_1) = -(x_1 - x_2) = -\frac{a_2}{Cm_1}$$

$$a_1 = -\frac{m_2}{m_1}a_2$$

This reassures us that the modification we made earlier works since we would expect the acceleration of one particle to be in the opposite direction of the other, which is the case. We have reached the stage which can tell us what can happen if we start plugging in values for m_1 and m_2. Now notice that if we were to change the mass of one of the particles to be negative, lets pick m_1, then let $m_2 = -\mu_2$ where $\mu_2 > 0$, and $m_1 = \mu_1 > 0$. We have the following,

$$a_1 = -\frac{(-\mu_2)}{\mu_1}a_2$$

$$a_1 = \frac{\mu_2}{\mu_1}a_2$$

Conclusion

The result above tells us that both of the accelerations are positive, since $\frac{\mu_2}{\mu_1} > 0$, and since they are both positive and we are on a line, it means that they are both accelerating in the same direction. We effectively have that in a perfect world these two particles will be accelerating together forever.